Maximum Drop Size Produced by Pneumatic Atomization

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In a study of the production of sprays from pneumatic atomizers, Kim and Marshall (1971) determined a generalized correlation for drop-size distribution by volume. The data points used for this correlation were read from their Figure 13 and are tabulated in the first two columns of Table 1. Here the drop-size is a normalized or reduced value obtained by dividing the actual diameter by the mass-median diameter in each case. These reduced sizes are correlated with Φ_v, the cumulative volume fraction less than a given size.

Kim and Marshall stated that these data "did not follow the usual distribution functions such as logarithmic, square root probability, or Rosin-Rammler." They fitted the curve by means of a Pearl-Reed or logistic equation (Pearl, 1930).

Because the data seem to show the possibility of an upper limit of drop size in the neighborhood of a reduced diameter of 3.0, and because there is no indication of a lower limit appreciably greater than zero, it seemed that the upper-limit function proposed by Mugele and Evans (1951) might be a suitable representation. According to this function, a value $u = \dfrac{X}{X_{max} - X}$, should be log-normally distributed with respect to Φ_v. Applied to the Kim and Marshall data, the X would be replaced by the reduced diameter $X^* = X/\overline{X}_m$ so that $u = \dfrac{X^*}{X^*_{max} - X}$ where $X^*_{max} = \dfrac{X_{max}}{X_m}$.

Mugele and Evans present an equation for obtaining an estimate of X_{max} from the data as follows:

$$\frac{X_{max}}{X_{50}} = \frac{X_{50}\,(X_{90} + X_{10}) - 2\,X_{90}X_{10}}{X^2_{50} - 2X_{10}}$$

From Table 1

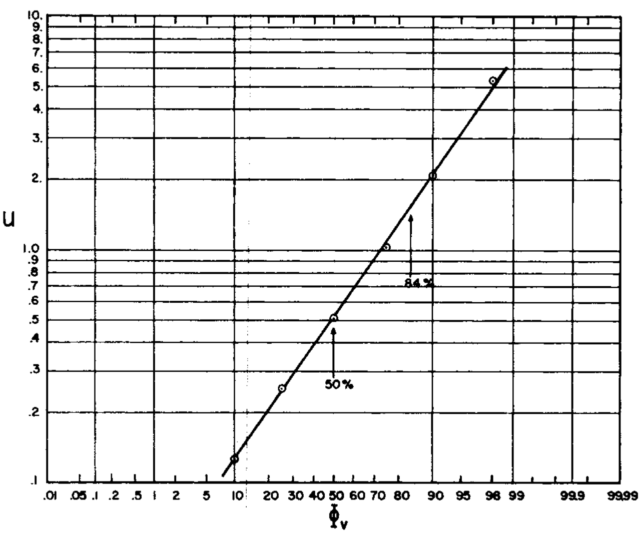

Fig. 1. Test of upper-limit function data of Kim & Marshall (1971).

$$X^*_{10} = 0.33 \quad X^*_{50} = 1.0 \quad X^*_{90} = 2.0$$

so that $X^*_{max} = 2.97$ from this equation.

The third column of Table 1 gives the values of u calculated from $X^*_{max} = 2.97$. These are plotted against Φ_v on log-normal probability paper, Figure 1, and give a good straight line, thus indicating a fit to the upper-limit distribution function. This line has a median value $u_{50} = 0.52$, and a geometric standard deviation $\sigma_g = u_{84}/u_{50} = 2.94$.

Values of Φ_v may thus be calculated from

$$\Phi_v = \frac{1}{2}\left[\,1 + \operatorname{erf}\frac{\ln u/0.52}{\sqrt{2}\,\ln 2.94}\,\right]$$

These are tabulated in the fourth column of Table 1 and agree well with the original data.

It is thus established that for sprays produced under any of the wide range of conditions and atomizer designs studied by Kim and Marshall the maximum drop-size will be very close to 3.0 times the mass-median size. Since Kim and Marshall give equations for predicting the mass-median drop size as a function of operating variables and nozzle design, it is possible to predict X_{max} likewise and to calculate a complete size distribution from the above equation for Φ_v. Alternatively, values of Φ_v may be read from Figure 1.

NOTATION

u $= X/X_{max} = X^*/X^*_{max} - X^*$
X = drop diameter, μm
X_{max} = maximum drop diameter in spray, μm
$\overline{X}_m$ = mass median drop diameter, μm
X^* = reduced drop diameter $= X/\overline{X}_m$
X_{10} = drop diameter corresponding to $\Phi_v = 0.10$
X_{50} = drop diameter corresponding to $\Phi_v = 0.50$
X_{90} = drop diameter corresponding to $\Phi_v = 0.90$

Greek Letters

σ_g = standard geometric dispersion of values of u
Φ_v = cumulative fraction by volume (or mass) of drops less than a specified size

Table 1. Upper-Limit Function Applied to Data of Kim and Marshall (1971)

Φ_v	X^*	$u = \dfrac{X^*}{2.97 - X^*}$	Φ_v calc.
0.0	0.0	0	0
0.10	0.33	0.125	0.093
0.25	0.60	0.253	0.252
0.50	1.0	0.508	0.491
0.75	1.5	1.02	0.734
0.90	2.0	2.06	0.899
0.98	2.5	5.32	0.984

LITERATURE CITED

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Boundary Condition at a Porous Surface Which Bounds a Fluid Flow

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A particular problem associated with the flow of a fluid over a porous surface is the selection of the appropriate boundary condition which must be used to give a realistic description of the tangential component of velocity at the permeable boundary. In order to allow for a nonzero velocity (that is, a slip velocity) at a porous boundary, Beavers and Joseph (1967) proposed a boundary condition in which the slip velocity is proportional to the velocity gradient at the boundary. The boundary condition involves an experimentally determined parameter which was argued to be independent of the fluid. Experiments with Poiseuille flows of oil and water through large aspect ratio rectangular ducts with one porous wall gave results which agreed with predictions based on the slip-flow boundary condition. Subsequent experiments with liquids in Poiseuille flows (Beavers et al., 1970; Sparrow et al., 1973) and Couette-type flow (Taylor, 1971) gave further support to the proposed boundary condition. Saffman (1971) has derived the form of the boundary condition analytically.

As yet, however, no investigations of the slip boundary condition in the presence of gas flows have been reported, and there has been no attempt to test the hypothesis that the slip parameter occurring in the boundary condition is independent of the fluid. The experiments described in this note were performed with the objective of establishing the validity of the slip boundary condition for gas flows and also of determining whether the fluid has a significant influence on the value of the slip parameter.

EXPERIMENTS

The experiments were performed in an open loop airflow facility. Air from a temperature-controlled laboratory room was drawn through a silica-gel dryer and a settling chamber into the test section. The test section was a large aspect ratio rectangular duct having a block of permeable material as the lower wall, an impermeable flat plate as the upper wall, and precisely machined spacer strips as the side walls. The smallest value of the duct height h was 0.24 mm (0.0097 in.) and the largest was 0.83 mm (0.0328 in.). The duct width was 8.9 cm (3.5 in.). Correspondingly, the cross-sectional aspect ratios ranged from about 110 to 360, thereby giving a close approximation to a parallel plate channel. The length of the test section was 40.6 cm (16 in.). The apparatus was designed so that the flows through the duct and the porous block were driven by the same axial pressure gradient.

Pressure gradients were recorded by means of taps which were axially distributed at 2.5-cm intervals along the center lines of both the upper wall of the duct and the lower wall of the test bed beneath the porous block. Pressure differences were read with a Baratron pressure gauge to as low as 0.01 mm mercury, and the measured pressure distributions along the upper and lower walls were found to be identical. The magnitude of the pressure gradient was chosen so that it was in the range for which a coupled parallel flow situation was established with fully-developed laminar flow in the channel and Darcy flow in the porous material. The two flows left the test section through separate exits, and each flow rate was measured by means of a rotameter.

The porous media consisted of two different specimens (Blocks A and B) of a material manufactured under the trade name of Foametal. This material consists of a homogeneous lattice-work of metal fibers such that there are no free fiber ends within the body of the material. Block A was 2.02 cm (0.795 in.) high and 32.5 cm (12.8 in.) long, while the corresponding dimensions for Block B were 2.25 cm (0.886 in.) and 37.5 (14.78 in.). The permeabilities were measured to be 6.67×10^{-7} cm^2 and 7.68×10^{-7} cm^2, respectively, for Blocks A and B. The porosities were on the order of 0.95, although they were not specifically measured for these experiments.

RESULTS

The slip condition at a permeable boundary is expressed by

$$(du/dy)_0 = (\alpha/k^{1/2})(u_0 - U) \qquad (1)$$

where α is the slip coefficient. The presence of a slip velocity affects both the friction factor and the mass flow in the duct. In the present experiments, measurements of friction factor (that is, pressure gradient) and mass flow were used to detect the effects of the slip velocity.

For fully-developed laminar flow in a parallel-plate channel with a slip boundary condition at one wall given by Equation (1), it can readily be shown (Beavers et al., 1970) that

$$\frac{C_f Re}{(C_f Re)^*} = \frac{m^*}{m} = \left[1 + \frac{3(\sigma + 2\alpha)}{\sigma(1 + \alpha\sigma)} \right]^{-1} \qquad (2)$$

where $\sigma = h/k^{1/2}$. Values of the $C_f Re$ product were calculated from the measured quantities for each experimental run, and the corresponding values of $(C_f Re)^*$ were computed from the theoretical solution for laminar flow in an impermeable-walled rectangular duct. The resulting values of the ratio $(C_f Re)/(C_f Re)^*$ for both porous blocks are plotted against Reynolds number for several values of $h/k^{1/2}$ in Figure 1. Inspection of Figure 1 shows that for a given $h/k^{1/2}$ there is no systematic dependence of $(C_f Re)/(C_f Re)^*$ with Reynolds number, which is consistent with the predictions of Equation (2). Further, since the data show that $(C_f Re)/(C_f Re)^* < 1$, there is a clear influence of velocity slip for all the duct heights used in these ex-